Coherent control of a qubit for a rotation gate operation in photonic crystals

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Abstract

We have developed a method of controlling the quantum state of an impurity atom embedded in photonic crystals that can be used as a single-qubit rotation gate, one of the universal quantum logic gates, using the coherent control of spontaneous emission from the embedded atom.

1. Introduction

The realization of quantum computers or communication is a challenging problem in micro- and nano-fabrication technologies. The fabrication of very small defects (such as quantum dots or impurity atoms) inside photonic crystals can provide promising building blocks in solid-state quantum-information-processing architectures. Near an embedded atom, light is confined due to the presence of a photonic bandgap (PBG), which leads to the formation of a photon-atom bound state that may provide the basis of qubit encoding and keeping quantum information [1]. Recent experiments have shown that spontaneous emission from light emitters embedded in photonic crystals is suppressed [2]. A recent major theoretical challenge in this photonic crystal system is the investigation of the quantum logic gate operation for processing quantum information [3]. To realize the quantum logic gate operation, a method of controlling the quantum state of a qubit should be developed. In particular, the control for realizing a single-qubit rotation gate, one of the universal quantum logic gates, is the key operation for quantum computing. So far, we have shown the coherent control of spontaneous emission from an impurity atom in photonic crystals [4]. Furthermore, we have demonstrated that light is confined in a single mode near an impurity atom in photonic crystals by the coherent control [5].

In this paper, we develop a method of controlling the quantum state of an impurity atom embedded in photonic crystals that can be used as a single-qubit rotation gate, using the results of our previous studies of coherent control of spontaneous emission from the embedded atom.

2. Qubit embedded in photonic crystals

A qubit is assumed to be composed of the two upper states ($|0\rangle$ and $|1\rangle$) of an impurity three-level atom embedded in photonic crystals with a high-pass 3D PBG [1]. One of the transition frequencies (between $|1\rangle$ and the ground state $|g\rangle$, ω_{1g}) is far inside the PBG, while the other transition frequency (between $|0\rangle$ and $|g\rangle$, ω_{0g}) is near the band edge frequency ω_c, where the detuning is denoted as $\delta = \omega_{0g} - \omega_c$. At the initial time ($t = 0$), using an ultrashort pumping laser pulse, the atom is prepared in the form

$$|\psi(0)\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle, \tag{1}$$

which can act as a qubit, where θ and φ are determined by the pulse area and phase of the pumping laser pulse. For $t > 0$, the control laser is used for coupling the bits $|0\rangle$ and $|1\rangle$, where the strength of the control laser is characterized by the Rabi frequency Ω.

The time evolution of the qubit

$$|\psi(t)\rangle = a_0(t)|0\rangle + a_1(t)|1\rangle \tag{2}$$

can be shown by a geometrical representation of a pure-state space composed of the two basis vectors $|0\rangle$ and $|1\rangle$, namely, the Bloch sphere, as shown in Fig. 1. The gray curve in Fig. 1(a) shows the qubit $|\psi(t)\rangle$ in the case where we neglect the effect of the PBG, that is, a free-space case. In this case, the norm of the qubit (the distance from the origin *O*) decays to zero due to atomic decay to the ground state $|g\rangle$, after a decaying random oscillation due to Rabi splitting by the control laser. Here, the rotation gate operation for a single qubit may be carried out by rotating the qubit on the circle *C*. Therefore, we clarify the approach to changing this time evolution to that on the circle *C* by dividing the approach into three steps using the results of our previous studies [4, 5]: step 1 where we suppress atomic decay, step 2 where we eliminate Rabi splitting and step 3 where we make this rotation approached the circle *C*.

3. Three steps for a single-qubit rotation gate operation

Step 1. In Fig. 1(b), we plot the time evolution of the qubit $|\psi(t)\rangle$, considering the effect of the PBG, which is given by calculating the quantum state of the embedded atom, using the results of our previous study [4]. In this case, spontaneous emission is suppressed, leading to the formation of a nondecaying photon-atom bound state, and then the qubit shows a nondecaying rotation. Here, the important parameter is the detuning δ. The strength of the suppression of atomic decay is increased by enhancing the detuning $|\delta|$, namely, by moving the transition frequency of the atom far inside the PBG. In Fig. 2, $\delta = -20\alpha^2$ with a scaled parameter α [1], which is in the range of realistic values and sufficient to suppress atomic decay while keeping the controllability of coupling the two bits $|0\rangle$ and $|1\rangle$ using the control laser.

Step 2. In Fig. 1(c), we apply the condition for making confined light near the atom composed of a single localized mode [5], where the initial atomic state $|\psi(0)\rangle$ is determined to form a destructive quantum interference in emission channels between the two upper states. For $\delta = -20\alpha^2$, the initial atomic state Eq. (1) is determined by the two phases $\varphi = 0.5\pi$ and $\theta = 1.2\pi$. Under this condition, Rabi splitting is eliminated due to the destructive quantum interference, so that the qubit $|\psi(t)\rangle$ immediately exhibits a circular rotation.

Step 3. Figure 1(d) shows the qubit $|\psi(t)\rangle$ in the long time limit ($t \to \infty$), for $\delta = -20\alpha^2$ determined in step 1, the two phases determined in step 2, and the various Rabi frequencies Ω that characterize the strength of the control laser. For a small Ω (see the C1 curve in Fig. 1(d): $\Omega = 2\alpha^2$, a weak-laser case), the qubit $|\psi(t)\rangle$ exhibits a circular rotation near the bit $|0\rangle$. On the other hand, this circle approaches the circle *C* by increasing Ω (see the C2 curve: $\Omega = 14\alpha^2$, a strong laser case). This is because, the control laser (Ω) provides the transition channel between the two bits $|0\rangle$ and $|1\rangle$, so that when Ω is increased, the population transfer between $|0\rangle$ and $|1\rangle$ is enhanced, and then both amplitudes $|a_{0,1}(t)|^2$ of the two bits approach each other. This provides a useful rotation for achieving a single-qubit rotation gate operation. However, in a stronger-laser case (C3 curve: $\Omega = 19\alpha^2$), the norm of the qubit is reduced. In this case, a large Stark shift is induced, so that the transition frequency of the atom moves to the band edge frequency from inside the PBG, leading to a reduction in the extent of suppressing the atomic decay. Consequently, we note that there are optimum Rabi frequencies Ω for achieving a single-qubit rotation gate operation. For example, in the case of $\delta = -20\alpha^2$, the Rabi frequency $\Omega = 14.9\alpha^2$ is given from an optimization to achieve a lower atomic decay with a higher $|a_0(t)|^2/|a_1(t)|^2$ ratio.

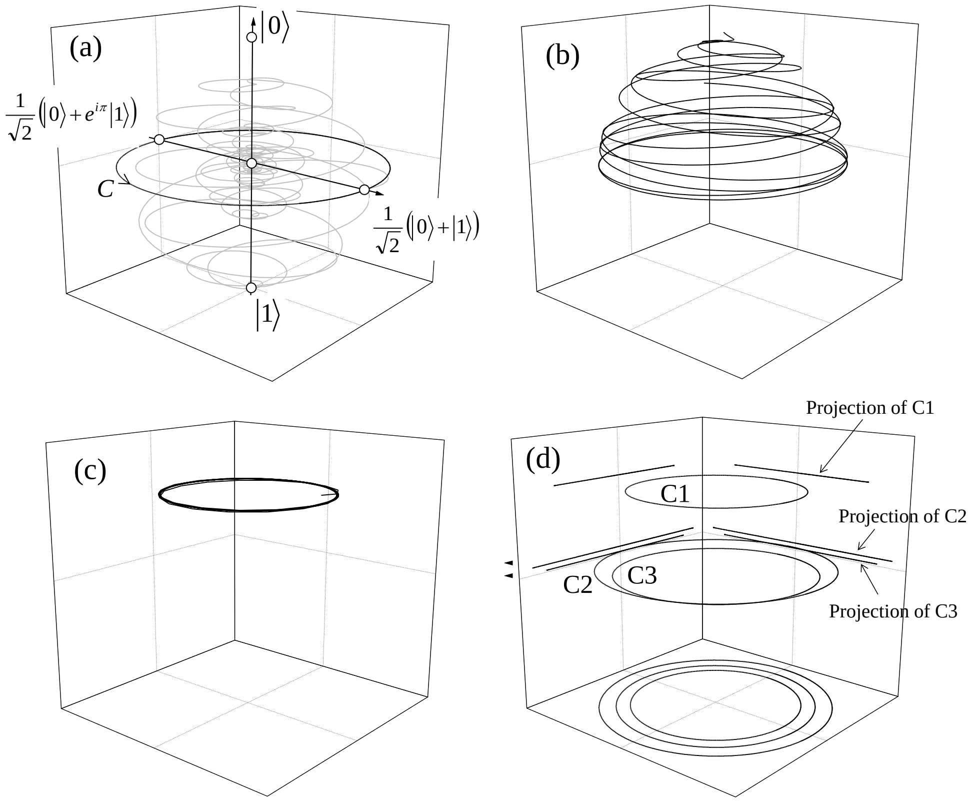

Fig. 1: Time evolution of qubit using Bloch sphere representation. (a) Free-space case, (b) PBG case where atomic decay is suppressed, (c) PBG case where Rabi splitting is eliminated and (d) PBG case for various Rabi frequencies where projections on xy-, yz- and zx planes are also plotted.

4. Conclusion

We have clarified a method of coherently controlling a qubit composed of two upper states of an impurity atom embedded in photonic crystals, which can be used for a rotation gate operation. First, we have determined the detuning between the transition and band edge frequencies, for the qubit to exhibit a nondecaying rotation in a pure-state space composed of the basis vectors of a qubit. Next, we have determined the initial atomic state that can be prepared by applying a pumping laser pulse onto the atom for the qubit to exhibit a circular rotation. Finally, we have found that there are optimum control laser strengths for operating a rotation gate for a qubit and determined the optimum control laser strength. The results of this study help us to determine the designs of future quantum logic gates based on photonic crystal systems.

References

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