Demonstration of flat band for terahertz coupled plasmon in metamaterials with kagomé symmetry

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Abstract

In this paper, we demonstrate the flat band for a coupled plasmon. A metamaterial composed of metallic discs and bars arranged to have kagomé symmetry is used. We theoretically show that a plasmonic flat band for the metamaterial is formed by the topological nature of the kagomé lattice. To confirm the flat-band formation, we make transmission measurements in the terahertz regime, and observe the flat band.

1. Introduction

From the aspect of geometric frustration in condensed-matter physics, the kagomé lattice has attracted considerable interest. Due to the frustration, the resonator system coupled to form the kagomé lattice has extensive degeneracy of nondispersing resonant modes[1]. These wavefunctions are trapped by the topology of the kagomé lattice. Because they are not coupled with each other, the eigen frequency is independent of wavevectors. Thus, the flat band is formed in the kagomé lattice. It is interesting that the flatness of the band is caused by interference effect in spite of the presence of coupling.

The formation of electromagnetic flat bands in kagomé lattices has already been predicted theoretically in some electromagnetic systems, such as two-dimensional photonic crystals[2] and metallophotonic waveguide networks[3]. In the flat band, the group velocity is slowed down in all directions and the effective mass of photons becomes very heavy. It is intriguing to study the flat band in metamaterials coupled with a kagomé symmetry, in terms not only of fundamental physics, but also from application standpoints, such as slow light. The circuit theory for flat-band formation is developed in [4]. In this paper, we experimentally demonstrate the electromagnetic flat band for a coupled plasmon in a metamaterial with kagomé symmetry.

2. Theory

In Fig. 1 (a), we introduce kagomé-type bar-disc resonators (KBDRs). Metallic discs are connected by narrow bars to form a kagomé lattice. Introducing the electric potential ϕ_i at the i-th disc and defining $\Phi_i := \int \phi_i \mathrm{d}t$, we obtain a Lagrangian $\mathcal{L}$ as

$$\mathcal{L} = \frac{C}{2}\sum_i \dot{\Phi}_i^2 + C_\mathrm{M}\sum_{i,j}\frac{A_{ij}}{2}\dot{\Phi}_i\dot{\Phi}_j - \frac{1}{2L}\sum_{i,j}\frac{A_{ij}}{2}(\Phi_i - \Phi_j)^2, \tag{1}$$

with capacitance C of the disc, inductance L of the bar, coefficient of electric induction C_M between nearest discs, and adjacency matrix A_{ij} of the kagomé lattice, whose element is 1 if i-th and j-th discs are directly connected by a bar for $i \neq j$; otherwise 0. The first, second, and third terms of (1) represent the electric energy stored on discs, mutual electric energy stored between discs, and magnetic energy stored around bars, respectively.

From the Euler–Lagrange equation $(\mathrm{d}/\mathrm{d}t)(\partial\mathcal{L}/\partial\dot{\Phi}_i) - \partial\mathcal{L}/\partial\Phi_i = 0$, coupled charge equations are obtained as:

$$\ddot{q}_i + \omega_0^2(4q_i - \sum_j A_{ij}q_j) + \kappa\sum_j A_{ij}\ddot{q}_j = 0, \tag{2}$$

with stored charge $q_i = C\dot{\Phi}_i$ at the i-th disc, resonant angular frequency $\omega_0 = 1/\sqrt{LC}$, and coupling coefficient $\kappa = C_{\mathrm{M}}/C$. In the frequency domain, (2) is simplified as

$$\sum_j A_{ij}\tilde{q}_j = \frac{4-(\omega/\omega_0)^2}{1+\kappa(\omega/\omega_0)^2}\tilde{q}_i, \tag{3}$$

where tildes represent complex amplitudes. Owing to the translational invariance of lattice, (3) can be reduced to an eigenvalue problem for a 3×3 matrix. Solving the eigenvalue problem, we obtain the dispersion relation for three bands as

$$\frac{\omega}{\omega_0} = \sqrt{\frac{6}{1-2\kappa}},\ \sqrt{\frac{3+2(3+F)\kappa \pm (1+4\kappa)\sqrt{3+2F}}{1+2\kappa-2(1+F)\kappa^2}}, \tag{4}$$

where $F = \cos\boldsymbol{k}_\parallel\cdot\boldsymbol{a}_1 + \cos\boldsymbol{k}_\parallel\cdot\boldsymbol{a}_2 + \cos\boldsymbol{k}_\parallel\cdot(\boldsymbol{a}_1-\boldsymbol{a}_2)$ with wavevector $\boldsymbol{k}_\parallel$ in the xy-plane and unit-lattice vectors $\{\boldsymbol{a}_1,\ \boldsymbol{a}_2\}$ shown in Fig. 1 (a). A calculated dispersion relation is shown in Fig. 1 (b) for $\kappa = 0$. The highest band $\omega/\omega_0 = \sqrt{6/(1-2\kappa)}$ is flat or independent of $\boldsymbol{k}_\parallel$. The lowest band shows conical dispersion near the Γ point, and the bending middle band touches the flat band at the Γ point.

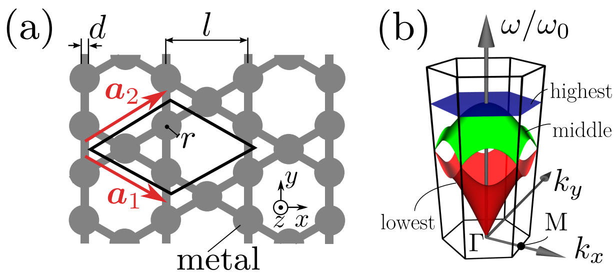

Fig. 1: (a) Schematic view of kagomé-type bar-disc resonators (KBDRs). (b) Dispersion relation of KBDRs for $\kappa = 0$.

3. Experiments

In order to study the dispersion relation of KBDRs, we conduct THz time-domain spectroscopy (THz-TDS), shown in Fig. 2 (a). We fabricate KBDRs on a stainless steel plate. The dimensions depicted in Fig. 1 (a) are: period between bars $l = 800\,\mu$m, bar width $d = 10\,\mu$m, disc radius $r = 145\,\mu$m, and metal thickness $h = 30\,\mu$m. The sample is rotated by θ with respect to the y-axis from normal incidence. The angles θ range from $\theta = 0°$ to $\theta = 65°$ with a step $\Delta\theta = 2.5°$. The magnitude of the wavevector $\boldsymbol{k}_\parallel$ on the sample plane is given by $k_\parallel = (\omega/c)\sin\theta$, where c is the speed of light. We perform transmission experiments for two polarizations along the x'-axis (parallel configuration) and y-axis (perpendicular configuration).

Figure 2 (b) displays the transmission spectrum for parallel configuration. Transmission spectrum minima are observed from 0.21 THz to 0.28 THz. With an increase of wavenumber, the frequency of the transmission minimum decreases from 0.28 THz at the Γ point and approaches 0.21 THz at the M point. This band corresponds to the second band of KBDRs. Surface charge distribution at the white circle is calculated by a commercial finite element method solver (Ansoft HFSS), and displayed in the figure.

By using (4), the fitting parameters are obtained from experimental data as $\omega_0/(2\pi) = 0.105\,\text{THz}$ and $\kappa = 0.103$.

Figure 2 (c) displays the transmission spectrum for perpendicular configuration. Unlike in the case of parallel configuration, the flat band of the transmission minima is observed around $0.28\,\text{THz}$. Surface charge distribution at the white circle is calculated by HFSS and displayed in the figure. The highest band given by (4) with the previously derived parameters is shown as a dotted line in Fig. 2 (c). It fits well the minima experimentally obtained.

4. Conclusion

We studied the electromagnetic flat band on a metallic kagomé lattice. A dispersion relation composed of three bands was theoretically predicted for kagomé-type bar-disc resonators. We experimentally observed two bands formed by transmission minima depending on the polarization of the incident terahertz beams. One of the bands corresponded to the flat band. This is the first experimental demonstration of the flat band for a metamaterial with kagomé symmetry in spite of existence of coupling[5].

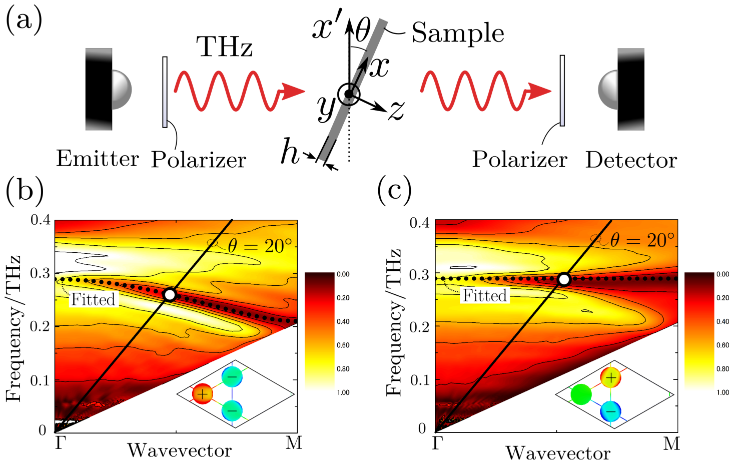

Fig. 2: (a)Schematic view of the transmission experiment. The plane of a sample is represented by the coordinate system (x, y) shown in Fig. 1 (a). (b) Experimentally obtained transmission diagram of KBDRs for parallel configuration (Polarization $\boldsymbol{E}$ is parallel to the x' axis). Transmission minima are fitted by dotted curve. Calculated surface charge distribution at the white circle is displayed in the figure. (c) Experimentally obtained transmission diagram of KBDRs for perpendicular configuration ($\boldsymbol{E}$ is parallel to the y axis). The flat band formed by transmission minima is observed. Calculated surface charge distribution at the white circle is displayed in the figure.

References

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