Analysis of the nanolaser linewidth using semiclassical laser model

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Abstract

Stochastic characteristics (radiation linewidth) of the nanolasers (such as the spaser and the dipole nanolaser), are investigated analytically and numerically. In frame of the semiclassical model gain dynamics is treated quantum mechanically based on density matrix approach, while the plasmon dynamics and radiation are assumed to be classical. It is shown that direct application of Schawlow-Townes overestimates the linewidth and has to be avoided at the description of the nanolaser. Analytical results are compared with ones obtained from the direct numerical simulation of the dynamics of the laser equations. The features of the below-, near- and above-threshold operation regimes are considered.

Introduction

Thanks to their unique optical properties, coherent radiation sources play a detrimental role in fundamental and applied areas of science. High-precision measurements, extreme energy physics and medicine are just several examples. Nowadays the field of nanooptics opens up fascinating perspectives and is of incredible attention. In particular, compact devices based upon nanoplasmonic effects are of great interest for researchers. Among of many suggested applications of plasmonic components lasing nanodevices play a special role, since their tiny volumes benefits in compactness exhibit novel physical phenomena. The plasmonic nanolaser (spaser) has been first proposed by Bergman and Stockman [1]. In this device the atoms or molecules of the gain media are coupled to a metallic nanoparticle, supporting surface plasmons, in such a way that pumping yields oscillations of the plasmons. Protsenko et al [2] proposed the dipole nanolaser, where the dipole moment of the nanoparticle is induced, which is basically one of the possible of description of the plasmons. In 2009 the spaser generation has been first demonstrated by Noginov et al [3].
In laser physics the traditional approach for definition of the laser linewidth is to use the known Schawlow-Townes (ST) formula. This formula, however, can be applied only for lasing systems that have the polarization relaxation time much shorter than the population and the optical resonator relaxation times (inversely proportional to the quality factor of the resonator). For the plasmonic based nanolasers the plasmon relaxation time is of the order of the polarization relaxation time, so that the conditions for the ST formula are not fulfilled. However, some authors have used the ST formula unjustified when considering the nanolasers, though obtaining values agreed with the experiment (in particular [4]). Therefore, a generalization of the ST formula is required. For the above-threshold regime the nanolaser linewidth calculation has been done for the so called bad-cavity lasers (see, for example, [5]). The generalized analytical expression for the linewidth calculation has been recently obtained in [6].

ISBN 978-952-67611-2-1

The aim of the report is to analyse the linewidth of the nanolaser operating at different regimes and compare the results with the known experimental data.

Description of the used model

For the analysis of the spectral linewidth of the radiation, generated by a plasmonic nanolaser we exploit the semi-classical approach, in which the gain medium is treated quantum-mechanically, while the electromagnetic field and plasmon dynamics are assumed to be classical. We assume also that the external electromagnetic field is resonant with the transition between two energy levels of the quantum system; the two level system model for the dynamics of the quantum ingredient producing gain is accepted. We have, therefore, the following set of equations for the field amplitude A, the population difference D and the polarization of the gain p [6]:

$$\begin{aligned} &\dot{A} + \gamma A = -i2\pi\omega p + \xi_T(t), \\ &\dot{D} + \gamma_1 (D - D_0) = \frac{2i}{\hbar}(pA^* - p^* A), \\ &\dot{p} + \gamma_2 p = i\frac{|d|^2}{\hbar} AD + \xi_{SP}(t) \end{aligned} \tag{1}$$

where coefficient γ describes the field attenuation due to the losses in the nanoresonator in the absence of gain, γ_1 and γ_2 are the coefficients describing the population inversion and phase relaxation processes, d is the dipole moment associated with the lasing transition; parameter D_0 is the pumping parameter, which has the meaning of population inversion in the absence of generation.
In (1) terms $\xi_{SP}(t)$ and $\xi_T(t)$ are the Langevin stochastic terms, taking into account the spontaneous emission and thermal fluctuations with the correlation functions:

$$\begin{aligned} &\left\langle \xi_{SP}^*(t)\xi_{SP}(t+\tau)\right\rangle = \gamma_2 D_{SP} e^{-\gamma_2|\tau|}, \\ &\left\langle \xi_T^*(t)\xi_T(t+\tau)\right\rangle = 2D_T\delta(\tau) \end{aligned} \tag{2}$$

where D_{SP} and D_T are parameters characterizing the intensity of the spontaneous emission and thermal noises respectively. The spontaneous emission and thermal fluctuations are the only sources due to which the nanolaser linewidth is broadened. Further, the reasonable assumption is to neglect the thermal fluctuations, since the spontaneous fluctuations contribution is dominant in the optical domain.

Results

Depending on the relation between the pumping parameter D_0 and the steady-state value of the population inversion D_{th} one can consider three operation regimes of the nanolaser: 1) the below threshold regime $D_0 < D_{th}$, 2) the near-threshold regime $D_0 \sim D_{th}$ and 3) the case of the above-threshold regime $D_0 > D_{th}$. The generalized Schawlow-Townes-like formula works only for the above-threshold regime (see [6]), so that the numerical analysis the other two cases, which can be realised in the experiments, is of primary interest.

The numerical study of the radiation linewidth of the stochastic system (1) was conducted by the second order Runge-Kutta stochastic method [7]. For this purpose the phase shift $\Delta\varphi(\tau) = \varphi(t) - \varphi(t-\tau)$ was considered, where $\varphi(t)$ is the phase of the slowly-varying component of the amplitude, appearing due to the stochastic forces.

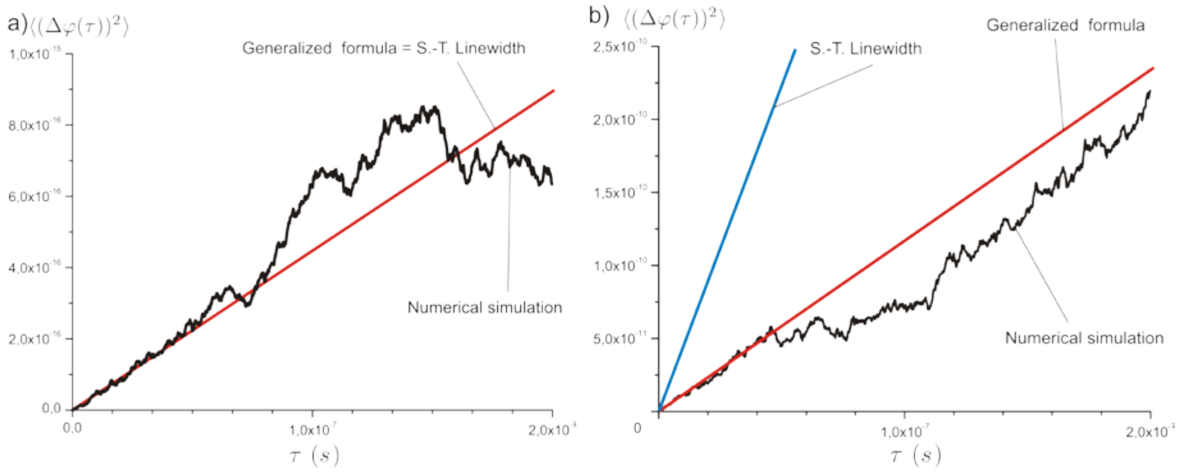

Fig.1: Analytical (red and blue curves) and numerical results (black curves) of the variance $\langle(\Delta\varphi(\tau))^2\rangle$ as a function of time delay. The curves are plotted for the following parameters: $\lambda = 0.532\ \mu m$, $|d| = 2.5\cdot 10^{-17}$ esu, $\gamma_1 = 10^9$ s^{-1}, $\gamma_2 = 10^{13}$ s^{-1}, $D_0 = 10^{17}$ cm^{-3}, $D_{SP} = 2\cdot 10^{-7}$. The resonator field attenuation rates are: a) $\gamma = 10^7$ s^{-1} which corresponds to ordinary laser with high quality resonator, and b) $\gamma = 10^{13}$ s^{-1} which corresponds to nanoresonator.

The above-threshold regime of the laser is the most convenient one, since the linewidth is predominantly determined by the phase fluctuations, while the amplitude ones are well-supressed. The power spectrum of the radiation is obtained through the Fourier transform of the autocorrelation function: $B(\tau) = \langle A^*(t)A(t-\tau)\rangle \sim \exp\left(-\frac{1}{2}\langle(\Delta\varphi(\tau))^2\rangle\right)$. In Figure 1 the numerical and analytical calculations of the variance of the phase shift $\Delta\varphi(\tau)$ is presented. Figure 1a corresponds to the ordinary laser with $\gamma << \gamma_1 << \gamma_2$, while Figure 1b – for the nanolaser with $\gamma \sim \gamma_2 >> \gamma_1$. As can be seen from Figure 1, the analytical and numerical results are in good agreement; the standard ST expression for the nanolaser overestimates significantly the more correct results.

The investigation of the below- and near-threshold regime turns out more subtle, which will also be presented in our talk.

References

[1] D.J. Bergman, M.I. Stockman, Surface plasmon amplification by stimulated emission of radiation: quantum generation of coherent surface plasmons in nanosystems, Phys. Rev. Lett., vol. 90, 027402, 2003.
[2] I.E. Protsenko, A.V. Uskov, O.A. Zaimidoroga, V.N. Samoilov, E.P. O'Reily, Dipole nanolaser, Phys. Rev. A, vol. 71, 063812, 2005.
[3] M.A. Noginov, G. Zhu, A.M. Belgrave, R. Bakker, V.M. Shalaev, E.E. Narimanov, S. Stout, E. Herz, T. Suteewong, U.Wiesner, Demonstration of a spaser-based nanolaser, Nature Letters, vol. 77, p. 1110, 2009.
[4] M.I. Stockman, The spaser as a nanoscale quantum generator and ultrafast amplifier, J. Opt., vol. 12, 024004, 2009.
[5] A.Z. Khoury, M.I. Kolobov, L. Davidovich, Quantum-limited linewidth of a bad-cavity laser with inhomogeneous broadening, Phys. Rev. A, vol. 53, № 2, p. 1120, 1995.
[6] A.S. Chirkin, A.V. Chipouline, Generalized expression for the natural width of the radiation spectrum of quantum oscillators, JETP Letters, vol. 93, № 3, p. 114, 2011.
[7] P.E. Kloeden, E. Platen, *Numerical Solution of Stochastic Differential Equations (Stochastic Modelling and Applied Probability)*, Springer, 2011.

