Tri-helical model for nanoplasmonic gyroid metamaterials

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Abstract

Metallic single gyroids are part of a novel class of self-assembled nanoplasmonic metamaterials exhibiting chiral behavior in the visible spectrum. To identify the physical origin of their chirality and quantify their electromagnetic properties, we develop an analytical model for a tri-helical metamaterial. Our model provides valuable insight into the optical properties of single nanoplasmonic gyroids, and the results show that the metamaterial's chirality is limited due to the structural integration of opposing helices along the main cubic axes, their connection into a continuous network - allowing induced current to flow - and the network's material properties, that is the dispersion of the metal.

1. Introduction

In 2004, it was theoretically shown that chirality in a resonant medium can lead to negative refraction for one circular polarization [1]. Following this, a negative refractive index was demonstrated in several chiral metamaterials such as Swiss Rolls [2] and 2D/3D helical wire arrays [3][4]. Until recently, the unique properties of artificial chirality were limited to microwave and even lower frequencies due to the complexity of top-down fabrication. However, developments in self-assembly and direct laser writing techniques now allow for the fabrication of chiral structures at the nanometer scale and with operating wavelengths in the visible. The most notable example is the gyroid [5][6], which is composed of several helices oriented along multiple directions and connected to each other. In order to comprehend the complex behavior of nanoplasmonic single gyroid (SG) metamaterials, we have developed an analytical model for a tri-helical metamaterial (THM), which is composed of three helices aligned with the three orthogonal axes, and use it as the basis to quantify the gyroid's chiral properties. Our model gives valuable insight into the physics of chiral metamaterials from microwave to the visible regime.

In this paper, we explain the physical mechanisms underlying the gyroid's chiral behavior on the basis of the THM model, supporting our conclusions with numerical simulation results on the THM and SG. We focus in particular on the polarization properties of longitudinal and transverse modes in both THM and SG. Finally, using simulation results for a gold gyroid approximated by the Drude model, we show the impact of realistic metals on the propagation of EM waves in the SG and fully characterize the SG's electromagnetic and chiral behavior.

2. Single Gyroid Metamaterials

Metallic gyroid metamaterials are nanoplasmonic composite structures that can be fabricated by self-assembly. The SG in particular has been considered as an excellent candidate for 3D optical chiral nanosized metamaterials. The metallic network of the SG metamaterial (shown in Fig. 1) derives from a triply-periodic minimal surface and can be approximated by

$$\sin x \cos y + \sin y \cos z + \sin z \cos x \geq t \tag{1}$$

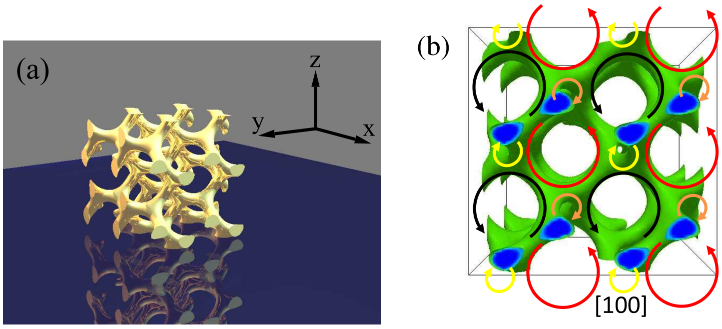

Fig. 1: (a) Gold SG metamaterial. (b) View of 2 × 2 × 2 SG unit cells along the [100] direction

where t determines the volume fraction. For our calculations, t is set to 1.2, corresponding to a volume fraction of 10%. As shown in Fig. 1 (b), the SG is composed of two left-handed helices (black and red arrows) and two right-handed helices with smaller radius (orange and yellow arrows) along the [100] direction of the cubic unit cell. Due to the symmetry of Equation 1, equivalent helices are found along the [010] and [001] directions.

3. Tri-Helical Model

To simplify the complex geometry of the SG, we assume three non-connected helices along the x, y, and z directions and arrange them in a cubic lattice of period a, as shown in the inset of Fig. 2(a). This arrangement leads to the tri-helical metamaterial, a 3D isotropic chiral medium whose effective electromagnetic parameters are identical along the major axes. Each helix of pitch p is created by wrapping a wire of thickness r_w around a cylinder of radius R.

The dispersion relation for transverse modes of the tri-helical metamaterial follows from [1]

$$\omega_{+\pm} = c_0 k_{+\pm}\left(i\kappa_{EH}^{-1}/c_0 \pm \sqrt{\chi_{HH}^{-1}\chi_{EE}^{-1}}\right) \quad \text{and} \quad \omega_{-\pm} = c_0 k_{-\pm}\left(-i\kappa_{EH}^{-1}/c_0 \pm \sqrt{\chi_{HH}^{-1}\chi_{EE}^{-1}}\right) \tag{2}$$

where χ_{EE}, χ_{EE} and κ are the effective electric permittivity, magnetic permeability and chirality[8]. Due to the tri-helical metamaterial's design, where one of the helices is oriented along the propagation direction, a longitudinal mode exist as well and obeys:

$$\omega = c_0\sqrt{A^2k^2 + k_p^2} = c_0\sqrt{\left(\frac{p}{l}\right)^2 k^2 + k_p^2} \tag{3}$$

where $l = \sqrt{(2\pi R)^2 + p^2}$ is the length of the wire for a given pitch, ε_d is the dielectric permittivity of the host medium and ω_p and ω_{0m} are the plasma and magnetic resonant frequencies respectively, given by:

$$\omega_p = c_0\sqrt{\frac{4\pi\tan^2\theta}{\varepsilon_d a^2\left(1 - \frac{\pi R^2}{a^2} + L_m\tan\theta\right)}} = \omega_{mp} \quad \text{and} \quad \omega_{0m} = c_0\sqrt{\frac{4\pi\tan^2\theta}{\varepsilon_d a^2\left(1 + L_m\tan\theta\right)}} \tag{4}$$

and $G = R\varepsilon_d/(2\tan\theta)$ is a constant, L_m is the self-inductance of the wires per unit cell and Γ and γ are loss parameters explained in more detail in Demetriadou *et al.* [8]. At optical wavelengths, Equation 4 remains unaffected, but we need to account for field penetration into the wires which affects L_m.

4. Numerical results

The band structure of the PEC gyroid metamaterial is shown in 2(b), where the lowest mode can been identified as the longitudinal mode and the two additional modes as transverse modes. A qualitative

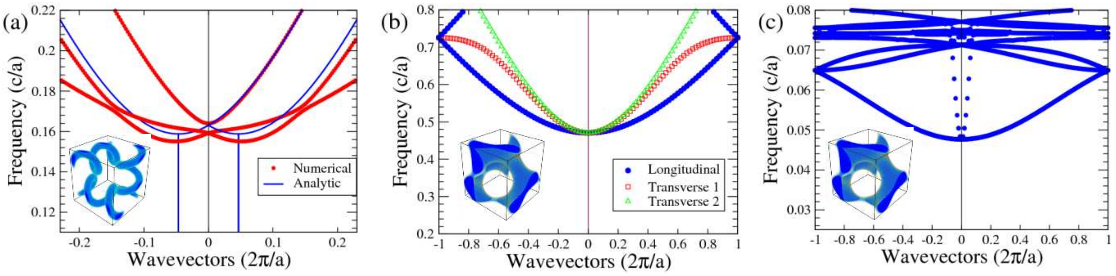

Fig. 2: The band structures along [100] direction for the (a) PEC tri-helical, (b) PEC gyroid and (c) gold gyroid metamaterial.

agreement with the band structure of the PEC tri-helical metamaterial is observed. The longitudinal mode is highly localized on the metallic wires of the gyroid and is slowed down due to chirality in accordance with results from the THM model. However, it is immediately evident that the mixture of right- and left-handed helices that compose the gyroid (shown in Fig. 2(b)) reduce its chirality [7][8] with the two transverse modes being degenerate for $k \to 0$. The band structure for a gold (Au) gyroid is shown in Fig. 2(c). By examining the electric and magnetic field mode profiles, the lowest branch is identified as a longitudinal mode, while the transverse modes exhibit a very steep slope indicative of a high group velocity. Furthermore, it is noteworthy that the effective plasma frequency of the gold gyroid is strongly reduced by a factor of 10 in comparison to the PEC SG. This is due to the increased skin-depth, which leads to a rise in the self-inductance and the effective mass of the electrons in the wires, causing the plasma frequency to be reduced. This effect is also observed in the tri-helical metamaterial and predicted by our analytical model.

5. Conclusions

The work presented here is the first step in identifying and quantifying the chirality in gyroids at infrared and visible frequencies. Despite having observed only small chirality for the single gyroid, we anticipate that the ability to explain their electromagnetic performance will lead to improved designs, where chirality can take a more prominent role.

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