Complementarity and Babinet principle in optical nano-circuits and metamaterials

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Abstract

Complementarity and Babinet principle are widely used concepts in optics and electronics. However, Babinet principle and complementarity can be only rigorously shown for infinitely thin perfect conducting screens. The extension of this proof to the optical range, where metals must be thick and characterized as negative permittivity dielectrics rather than as perfect conductors is not straightforward. Here, we explore the physics behind the generalization of Babinet and complementarity concepts to optical planar nano-circuits and metamaterials.

1. Theory

In this contribution we will explore the justification and the usefulness of the concepts of complementarity and Babinet principle in planar optical nano-circuits [1] - [3] and metamaterials [4], [5] operating in the optical range.

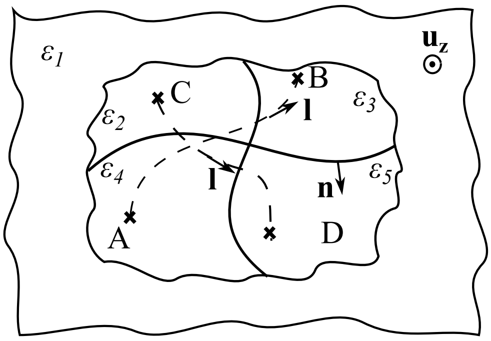

Fig. 1: Illustration of a planar nano-circuit like those analyzed in this contribution.

To begin with, let us consider the two-dimensional (2D) problem shown in Fig.1. It is a 2D piecewise homogeneous region filled by some media with relative (to vacuum) dielectric constants ε_i, which supports a quasi-electrostatic electric field $\mathbf{E} = -\nabla_t \phi(x, y) = -\mathbf{u}_x \partial_x \phi - \mathbf{u}_y \partial_y \phi$, where $\phi(x, y)$ satisfies Laplace's equation $\nabla_t^2 \phi = 0$. It is worth to mention that bound solutions for $\phi(x, y)$ may exist if there is at least one ε_i with $\mathrm{Re}(\varepsilon_i) < 0$, and at least one ε_j with $\mathrm{Re}(\varepsilon_j) > 0$. Assuming that this solution exists, the electric field inside each region must satisfy $\nabla_t \times \mathbf{E}_i = 0$ and $\nabla_t \cdot \mathbf{E}_i = 0$, as well as the boundary conditions at the border between i and j media $\mathbf{n} \times (\mathbf{E}_i - \mathbf{E}_j) = 0$ and $\mathbf{n} \cdot (\varepsilon_i \mathbf{E}_i - \varepsilon_j \mathbf{E}_j) = 0$, where $\mathbf{n}$ is the unit vector normal to this border and contained in the $x - y$ plane. The "complementary" structure

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is obtained by substituting in Fig.1 the permittivities ε_i by the "complementary" permittivities ε_i' defined by:

$$\varepsilon_i' = C_1/\varepsilon_i \tag{1}$$

where C_1 is an arbitrary constant. This definition of "complementarity" includes the conventional one as a particular case: when there are only two media and $C_1 = \varepsilon_1\varepsilon_2$.

The "complementary" fields $\mathbf{E}_i'$ inside each region of the complementary structure are defined by:

$$\mathbf{E}_i' = C_2\, \varepsilon_i\, \mathbf{u}_z \times \mathbf{E}_i \tag{2}$$

where C_2 is an arbitrary constant. It can be shown that these fields also satisfy the quasi-electrostatic equations for the complementary structure. Let be A, B, C and D some fixed points in the original and the complementary structures (see Fig.1). Let us define the voltage integral between A and B and the current integral through the path C-D as:

$$V_{AB} = \int_A^B \mathbf{E}\cdot\mathbf{l}\, dl \;\; ; \;\; I_{CD} = j\omega\varepsilon_0\, h \int_C^D \varepsilon(\mathbf{r})\mathbf{E}\cdot(\mathbf{u}_z \times \mathbf{l})\, dl \tag{3}$$

where h is the thickness of the circuit board. Let us assume that we can define some meaningful impedances $Z = V_{AB}/I_{CD}$ for the structure of Fig.1 and $Z' = V_{CD}'/I_{AB}'$ for its complementary one. By using (1) in (2) we obtain:

$$h^2k^2C_1\, Z\, Z' = -Z_0^2 \tag{4}$$

where $k = \omega\sqrt{\varepsilon_0\mu_0}$ is the phase constant and $Z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the vacuum impedance.

Let us now consider a diffraction screen made of a periodic planar nano-circuit board. Since nano-circuits must be electrically small in order to be described in the frame of quasi-electrostatics, the periodicity of the screen must be small too. Therefore, we can describe the screen as a surface impedance sheet. This surface impedance will be, in general, a 2D symmetric tensor whose main values are related with the nano-circuit impedances. Specifically, the surface impedance along a main axis of this tensor can be computed as $Z_s = Zl_{CD}/l_{AB}$ with Z defined above with the paths $A-B$ and $C-D$ chosen as straight lines going across the whole unit cell and directed along the proper main axes of the surface impedance tensor (l_{AB} and l_{CD} are the lengths of the corresponding paths). Specifically, this surface impedance Z_s describes the behavior of the screen for incident light polarized along the $A \to B$ direction. For the "complementary" screen (made of the complementary nano-circuit), the surface impedance for incident light of orthogonal polarization is $Z_s' = Z'l_{AB}/l_{CD}$, with Z' defined above. The transmission coefficient for the first screen and the considered incident light is:

$$t = \frac{2Z_s}{Z_0 + 2Z_s} \tag{5}$$

and the transmission coefficient for the complementary screen:

$$t' = \frac{2Z_s'}{Z_0 + 2Z_s'} = \frac{Z_0}{Z_0 + 2KZ_s} \;\; ; \;\; K = -h^2k^2C_1/4 \tag{6}$$

which for $K = 1$ reproduces the well known Babinet relation $t + t' = 1$ for perfect conducting complementary screens. If $K \neq 1$, Eqs. 5-6 still reproduce many of the main predictions of Babinet principle. Specifically, when $|Z| \to 0$, the transmittance $|t|^2$ has a zero, and according to (6), $|t'|^2$ has a maximum. For lossy media $|Z|$ never goes to zero, and the minimum of $|t|^2$ occurs at the minimum of $|Z|$, whereas the maximum of $|t'|^2$ occurs at the minimum of $|KZ|$. Since K is a function of frequency, this may lead to some deviation between the minimum of $|t|^2$ and the maximum of $|t'|^2$.

Eqs. 5-6 can be considered as the generalization of Babinet principle for planar nano-circuit screens. It is worth to mention, however, that these expressions are only approximate: they are valid in the quasi-electrostatic limit and as far as the effect of fringing fields can be neglected in order to calculate the total current in the screen. Since these constrains are approximately fulfilled by many planar nano-circuit boards and metallic metamaterial "atoms" operating at optical frequencies, this theory is expected to be useful for the analysis of these structures.

2. Results

In order to check the accuracy of our theory, it has been applied to the analysis of the structure shown in the inset of Fig.**??** (left). It is a 1D diffraction screen made of alternating layers of copper and silicon ($\varepsilon \approx 11.9$) which can be seen as the realization of an optical nano-circuit [6]. We first computed the transmittance through the screen by using the commercial solver CST Microwave Studio, and then obtained the transmittance through the complementary screen from (5)-(6) after elimination of the common variable Z. The results are shown in Fig.**??**, where a very good agreement between our theory and the electromagnetic simulations can be observed. The results coming from conventional Babinet principle ($t + t' = 1$) are also shown in the Figure. They show a significant deviation from the computed results, as it can be expected from the properties of the media involved in the screen. Our theory has been also applied to the computation of the transmittance through screens made of conventional and complementary SRRs operating in the optical range. Some results can be seen in Fig.**??** (right), where it can be appreciated how they approaches reality better than conventional Babinet principle ($t + t' = 1$), in spite of the fact that SRRs can not be considered as purely quasi-electrostatic entities except at very high frequencies, i.e. beyond saturation [7].

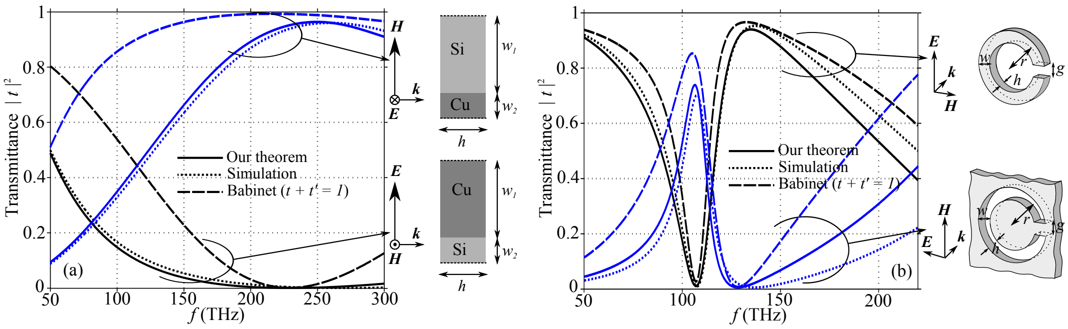

Fig. 2: Left: transmission through two complementary 1D diffraction screens made of alternating layers of copper and silicon (unit cells shown aside). Dimensions are $w_1 = 50$ nm, $w_2 = 10$ nm and $h = 25$ nm. Right: transmission through complementary silver SRR and CSRR screens with $r = 100$ nm, $g = 10$ nm, $w = 30$ nm and $h = 60$ nm. Periodicity is 250 nm.

3. Conclusion

Physical insight into complementarity at optical frequencies has been provided. Classical concepts of complementarity and Babinet principle have been extended to planar complementary nano-circuits and metamaterials. Results of our theorem have been validated with full wave electromagnetic simulation showing improved accuracy respect to standard Babinet principle.

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