Lasing in the photonic crystal

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Abstract

In this paper we describe the lasing in a 1D photonic crystal containing gain layers. It is shown that the lasing threshold can be determined by linear (negative loss) approximation. Connecting the onset of lasing with the passage of the transfer function pole into the upper half-plane of the complex frequency, we show that (i) if the pump frequency lies in a pass band then an increase in the number N of elementary cells will sooner or later lead to lasing; (ii) if the frequency of the pump lies in the band gap, then lasing at band gap frequencies may occur in a sample with a low N, lasing will be suppressed by a further increase in N.

1. Introduction

Recently, great interest has been revived in heterogeneous media, in which the interaction of electromagnetic fields with constitutive elements is of resonant character [1, 2]. Photonic crystals (PCs) providing resonance by their periodicity and metamaterials, in which the resonance is a feature of single inclusion, are examples of such media. The interest in such systems is caused both by new physical phenomena (band gaps for optical radiation [3], negative refraction [4, 5]) and possible applications.

However, in the majority of suggested applications the main limiting factor is the high level of loss [6]. To reduce the influence of losses the introduction of a gain medium into such systems was proposed [7]. Taking into account that the purpose of the gain medium application is the compensation of losses, which are described by imaginary parts of permittivity and permeability, one implies the possibility of treating the gain medium in terms of negative imaginary part of permittivity. This simple model has restrictions; in particular an increase in the total thickness of gain layers can lead to lasing. The linear approach describing gain as a negative loss, is considered, and the limits of its applicability resulting from the onset of lasing are determined.

2. Gain as negative loss

Generally, the description of light propagation in a gain medium requires the simultaneous solution of equations, taking into account the interaction of quantum-mechanical states of an excited atom (quantum dot) with an electromagnetic field [8, 9]. In the presence of a large number of photons in the electromagnetic modes considered, the field can be described by the classical Maxwell equations [8]. At the same time, the behavior of a two-level system, modeling the quantum dot, requires a quantum-mechanical description. Such an approach leads to the system of Maxwell–Bloch equations [9]. This model is non-linear and describes stationary and dynamic lasing regimes.

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However, below the lasing threshold we can confine ourselves to weak fields and apply a linear approach, reducing the Maxwell–Bloch equations to the Helmholtz equation with permittivity following the ‘‘antiresonant’’ Lorentz dispersion law (1)

$$\varepsilon = \varepsilon_0 - \frac{\alpha}{\omega_0^2 - \omega^2 - 2i\gamma\omega} \tag{1}$$

where ω_0 is the pumping frequency, γ is the line width, α is proportional to the population inversion and is determined by the pump intensity [10, 11].

It is worth emphasizing that such a dispersion law (1) with positive values of both α and γ satisfies the Kramers–Kronig relation and the causality principle (all poles of permittivity are in the bottom half-plane of a complex frequency). Since the imaginary part of permittivity is negative, the gain may be considered as a negative loss. This approach is only valid in the absence of lasing, as the latter is essentially a non-linear regime. Nevertheless, the condition of onset of lasing may be obtained in the linear approximation.

3. Lasing in photonic crystal

Thus, we consider a finite sample of one-dimensional PC. The sample includes N periods, each consisting of a dielectric layer with permittivity $\varepsilon_1 = 1$ and a gain layer with permittivity $\varepsilon_2 = 1$, determined by (1). These layers are of thicknesses d_1 and d_2, respectively. Under linear approximation, the T-matrix method is usually applied to calculate the transmission and reflection coefficients.

It is known that the causality principle constrains analytical properties of the transmission and reflection coefficients. In the case of normal incidence and a finite number of layers, all the irregularities of the transmission and reflection coefficients are poles, corresponding to the eigenmodes of the PC sample considered as an open resonator. The complex eigenfrequencies are determined by the divergence of the amplitudes the transmission and reflection coefficients, which are expressed through the T-matrix elements T_{ij} as $t = 1/T_{22}$, $r = -T_{12}/T_{22}$ [12-14]. Thus, the position of the poles can be found from the equation

$$T_{22}(\omega) = 0 \tag{2}$$

In the absence of gain, all the poles of the transmission and reflection coefficients (2) are in the lower half-plane of the complex frequency.

In the case of a gain medium, additional poles of appear (2). At large enough gain they may occur in the upper half-plane. Appearance of the poles in the upper half-plane should lead to the violation of the causality and the beginning of lasing.

Our calculations show that an increase of the pump shifts the poles upwards, irrespective of whether they are in the pass band or in the band gap. The vertical shift of the poles with the increase of the number of cells N in the PC sample depends on the position of the poles. In the pass band they move upwards, whereas in the band gap they move downwards at the complex frequency plane [15]. This shift is manifested as an amplification of the transmission resonances in the band and as a suppression of them in the band gap with an increase in N. In the latter case lasing may be observed at small N when the band gap has not been formed, whereas at large N the band gap effect suppresses lasing.

4. Conclusion

If the pump exceeds the lasing threshold, direct application of the T-matrix method yields incorrect results, even if the calculated fields are small enough to satisfy the linear approach. This is due to the fact that, generally, the lasing frequency is not that of the incident wave, but corresponds to the position of the pole. The behavior of the poles when the pump is in the pass band of the PC [16] is analogous to the case of the homogeneous layer: with an increase in the total thickness of the gain layers these poles cross the real frequency axis in the upward direction, leading to the onset of lasing. The poles in the band gap, on the other hand, can only produce lasing at a small enough number of layers. This situation is similar to the oblique incidence of light on a homogeneous gain layer at angles providing total reflection [17].

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