Electromagnetic phenomena in omega nihility media

Y. Ra'di and S. A. Tretyakov

Department of Radio Science and Engineering, School of Electrical Engineering, Aalto University
P.O. Box 13000, FI-00076, Aalto, Finland
email: younes.radi@aalto.fi

Abstract

Omega material is a reciprocal bi-anisotropic material with an antisymmetric magnetoelectric dyadic (e.g., a composite formed by conductive Ω-shaped particles). In this presentation we will discuss the extreme electromagnetic properties of omega materials in the limiting case of *nihility*, when both permittivity and permeability of the medium tend to zero, and the magnetoelectric parameter alone defines the material response. Among other effects, we show that the omega nihility material provides the extreme asymmetry in reflection from a material slab: The reflection coefficients from the two opposite sides differ by sign, while the transmission coefficient is symmetric as in any reciprocal-material slab.

1. Introduction

It is well known that the shape and dimensions of chiral particles (for example, helices) can be optimized in such a way that these optimal helices radiate waves of only one circular polarization, whatever is the exciting field (e.g. [1, 2]). The difference of the propagation factors β of the two (circularly polarized) eigenwaves in isotropic chiral media is defined by the chirality parameter κ in the Tellegen formalism as $\beta = k_0(n \pm \kappa)$, where $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the free-space wavenumber and $n = \sqrt{\epsilon\mu/(\epsilon_0\mu_0)}$ is the refractive index. In the extreme case when $n \to 0$ but $\kappa \neq 0$, mixtures of optimal spirals realize effective media with the propagation factor $\beta = -k_0$ for one of the circular polarizations, while the same medium is transparent for the orthogonal circular polarization [3]. This extreme-parameter medium is called *chiral nihility* [4]. The concept of chiral nihility lead to understanding of the chiral route to negative refraction and superlensing with the use of chiral structures. In the contents of this work, we can say that the asymmetry in the propagation constants of the two eigenwaves comes to its extreme (the two values differ by sign) in the limiting case of chiral nihility. The wave impedances of the two eigenwaves are always the same as they do not depend on the chirality parameter κ.

More recently, it was understood that there are optimal parameters (and the optimal shapes and sizes) also for the other fundamental class of reciprocal bi-anisotropic media: Omega materials [5]. In this case the magnetoelectric dyadic is antisymmetric and the material relations take the form (uniaxial material, the axis along $\mathbf{z}_0$)

$$\mathbf{D} = \overline{\overline{\epsilon}} \cdot \mathbf{E} + j\sqrt{\epsilon_0\mu_0}\, K\mathbf{z}_0 \times \mathbf{H}, \qquad \mathbf{B} = \overline{\overline{\mu}} \cdot \mathbf{H} + j\sqrt{\epsilon_0\mu_0}\, K\mathbf{z}_0 \times \mathbf{E} \tag{1}$$

Here, $\overline{\overline{\epsilon}} = \epsilon_0(\epsilon_t\overline{\overline{I}}_t + \epsilon_n\mathbf{z}_0\mathbf{z}_0)$, $\overline{\overline{\mu}} = \mu_0(\mu_t\overline{\overline{I}}_t + \mu_n\mathbf{z}_0\mathbf{z}_0)$, and $\overline{\overline{I}}_t$ is the two-dimensional unit dyadic. The material parameter K measures the strength of magnetoelectric coupling and it is called the omega parameter.

In contrast to chiral media where the chirality breaks the symmetry of the propagation constants of eigenwaves while the wave impedances are not affected, in omega materials the properties are dual: Magneto-electric coupling breaks the symmetry of the wave impedances while the propagation constants remain symmetric. We expect that *omega nihility* media may have extreme properties similar (but dual) to those in the chiral nihility media.

The goal of this work is to study fundamental properties of omega materials in the case when the permittivity and permeability are negligibly small as compared with the magneto-electric coupling coefficient K and understand what kind of extreme wave properties can be expected for such special materials.

2. Omega nihility

For the uniaxial omega media the Maxwell equations after two-dimensional Fourier transform in the transverse plane take the form [6]

$$\begin{aligned} -j\mathbf{k}_t \times \mathbf{E} + \frac{\partial}{\partial z}\mathbf{z}_0 \times \mathbf{E}_t &= -j\omega\bar{\bar{\mu}} \cdot \mathbf{H} + k_0 K \mathbf{z}_0 \times \mathbf{E}_t \\ -j\mathbf{k}_t \times \mathbf{H} + \frac{\partial}{\partial z}\mathbf{z}_0 \times \mathbf{H}_t &= j\omega\bar{\bar{\epsilon}} \cdot \mathbf{E} - k_0 K \mathbf{z}_0 \times \mathbf{H}_t \end{aligned} \tag{2}$$

Here, $\mathbf{k}_t$ is the transverse component of the wave vector and the axis of the uniaxial material is along the unit vector $\mathbf{z}_0$.

Let us assume that $\epsilon_{t,n}$, $\mu_{t,n}$ are negligibly small. To investigate wave solutions in this case, one approach is to simply set $\epsilon_{t,n}$, $\mu_{t,n}$ to be zero and solve the field equations. Setting $\epsilon_{t,n}$ and $\mu_{t,n}$ to zero in Eq. 2, we get

$$\mathbf{k}_t \times \mathbf{E}_t = 0, \quad \mathbf{k}_t \times \mathbf{H}_t = 0 \tag{3}$$

and

$$-jE_n\mathbf{k}_t - \left(\frac{\partial}{\partial z} - k_0 K\right)\mathbf{E}_t = 0, \quad -jH_n\mathbf{k}_t - \left(\frac{\partial}{\partial z} - k_0 K\right)\mathbf{H}_t = 0 \tag{4}$$

As it is seen from Eqs. 3 and 4, there are no propagating waves in this medium. But, what will happen when ϵ and μ tend to zero with possibly different rates? Will the results be the same? It appears that when we set the permittivity and permeability to be absolutely zero from the beginning, some possible solutions are suppressed. Considering this, one can solve the equations in the general case and then investigate the results when the parameters go to zero with different rates. Now, using the known solution for wave propagation in the general bi-anisotropic case [6], we want to consider the propagation wave behaviour when the parameters go to zero at different rates. We start with the case in which the permittivity and permeability approach zero as linear functions possibly with different slopes, writing

$$\epsilon_t = e_t l, \quad \epsilon_n = e_n l, \quad \mu_t = m_t l, \quad \mu_n = m_n l, \quad \text{and} \quad l \to 0 \tag{5}$$

where e_t, e_n, m_t, and m_n are constants. Parameter l can be, for example, the frequency shift from the nihility point. Substitution of a plane-wave solution in the form $\exp(-j\beta z)$ in Eq. (2), the propagation factor β can be solved as

$$\frac{\beta^2}{k_0^2} = -K^2 - \frac{k_t^2}{k_0^2}\frac{1}{2e_n m_n}[(e_t m_n + e_n m_t) \pm (e_t m_n - e_n m_t)] \tag{6}$$

We see that the resulting dispersion equation depends on the rates with which different components of the permittivity and permeability dyadics approach zero. In particular, if $\epsilon_{t,n}$ and $\mu_{t,n}$ tend to zero with different rates, it is possible to realize propagating or evanescent wave regime in the lossless medium. An exception is the case of axial propagation ($k_t = 0$, or propagation along the z axis), where the result is same for arbitrary slopes of the linear functions (5):

$$\frac{\beta^2}{k_0^2} = -K^2 \tag{7}$$

The waves along the axis are evanescent in the lossless case.

The wave impedances for the eigenwaves propagating along the axis ($k_t = 0$) read

$$Z_{\pm}^{TM} = Z_{\pm}^{TE} = \lim_{l \to 0} \left[j \frac{1}{e_t} \sqrt{\frac{\mu_0}{\epsilon_0}} \left(K \pm K \right) \frac{1}{l} \right] \tag{8}$$

As we see, the asymmetry in the wave impedances for oppositely-bound waves indeed goes into extreme: For one direction we have $Z_{+}^{TM} = Z_{+}^{TE} = j\infty$ and for the opposite direction $Z_{-}^{TM} = Z_{-}^{TE} = 0$.

Next we consider a slab filled with an omega nihility material (the axis is normal to the interfaces) in free space under illumination of a normally incident plane wave. Reflection and transmission coefficients can be easily calculated using the known vector-transmission-line model with different wave impedances for eigenwaves travelling in the opposite directions [6, 7]. This gives the following simple formulas:

$$R_{\pm} = \pm \frac{-1 + e^{-2j\beta d}}{1 + e^{-2j\beta d}} = \mp j \tan(\beta d), \qquad T = \frac{2e^{-j\beta d}}{1 + e^{-2j\beta d}} = \frac{1}{\cos(\beta d)} \tag{9}$$

The two signs in the reflection formula correspond to the illumination from the opposite sides of the slab. As expected, the asymmetry in the reflection properties goes into extreme: The reflection coefficients differ by sign. The transmission coefficient is, of course, symmetric as in any reciprocal structure. Here we have assumed that losses are negligible. In this case the propagation constant $\beta = \pm jKk_0$ is purely imaginary (7), and it is easy to check that the energy conservation relation $|R_{\pm}|^2 + |T|^2 = 1$ is identically satisfied. In the limit of $d \to \infty$ (half-space) we see that the reflection coefficient tends to ± 1 depending on the sign of the coupling coefficient. Thus, reversing the sign of K, we have either an electric or magnetic wall behaviour at the interface.

3. Conclusion

Exotic properties of omega nihility have been considered. It was found that the omega nihility material exhibits extreme asymmetry in the wave impedances for oppositely bound waves. Reflection coefficients from a slab constructed from this material differ by sign for illumination from the opposite sides. It was found that the rate in which ϵ and μ tend to zero is in most cases a factor which determines the electromagnetic response of the composite.

References

[1] I.V. Semchenko, S.A. Khakhomov, A.L. Samofalov, Radiation of circularly polarized electromagnetic waves by the artificial flat lattice with two-turn helical elements, *Bianisotropics2004,* 10th International Conference on Complex Media and Metamaterials, Ghent, Belgium, pp. 236239, 22-24 September 2004.

[2] I.V. Semchenko, S.A. Khakhomov, A.L. Samofalov, Optimal shape of spiral: Equality of dielectric, magnetic and chiral properties, *NATO ARW & META'08,* Marrakesh, Morocco, pp. 71-80, 7-10 May 2008.

[3] I.V. Semchenko, S.A. Khakhomov, and S.A. Tretyakov, Chiral metamaterial with unit negative refraction index, *The European Physical Journal - Applied Physics,* vol. 46, no. 3, p. 32607, 2009.

[4] S. Tretyakov, I. Nefedov, A. Sihvola, S. Maslovski, C. Simovski, Waves and energy in chiral nihility, *Journal of Electromagnetic Waves and Applications,* vol. 17, no. 5, pp. 695-706, 2003.

[5] S.A. Tretyakov, The optimal material for interactions with linearly-polarized electromagnetic waves, Proc. of the *Fourth International Congress on Advanced Electromagnetic Materials in Microwaves and Optics (Metamaterials'2010)*, pp. 65-67, Karlsruhe, Germany, September 13-18, 2010.

[6] S.A. Tretyakov and A.A. Sochava, Reflection and transmission of plane electromagnetic waves in uniaxial bianisotropic materials, *Int. J. of Infrared and Millimeter Waves,* vol. 15, pp. 829-856, 1993.

[7] A.N. Serdyukov, I.V. Semchenko, S.A. Tretyakov, A. Sihvola, Electromagnetics of bi-anisotropic materials: Theory and applications, Amsterdam: Gordon and Breach Science Publishers, 2001.