Non-birefringent omega-like media

Sérgio A. Matos[1], Filipa R. Prudêncio[2], Carlos R. Paiva[2]

[1] Instituto de Telecomunicações, Department of Information Science and Technology, ISCTE – University Institute of Lisbon.
Avenida das Forças Armadas, 1649-026 Lisboa, Portugal
Email: sergio.matos@lx.it.pt
[2] Instituto de Telecomunicações, Instituto Superior Técnico, Technical University of Lisbon.
Av. Rovisco Pais, 1, 1049-001 Lisboa, Portugal
Email: fprudencio@lx.it.pt, carlos.paiva@lx.it.pt

Abstract

Birefringence is a common effect in anisotropic, bi-isotropic and bianisotropic media. Nevertheless, under strict restrictions on the constitutive parameters, these media can be non-birefringent as well. Recently, non-birefringent media (NBM), especially the anisotropic case, have attracted some attention in the literature. In this work we focus on metamaterials composed of omega inclusions, namely: omega and pseudochiral omega media. We show that omega media can be non-birefringent. Moreover, for pseudochiral omega media, a different kind of NBM was found, that we dubbed as two-dimensional non-birefringent media (2D-NBM). NBM are characterized by a single isofrequency surface, whereas 2D-NBM have two isofrequency surfaces that intersect in a plane.

1. Introduction

Recently non-birefringent media (NBM) became a topic of interest for the metamaterial community namely in relation with transformation optics [1]. In fact, anisotropic NBM were already addressed in the literature with different designations: singly-refractive media [2], pseudo-isotropic [3], quasi-isotropic [4]. Moreover, bianisotropic NBM are an important topic in premetric electrodynamics. In [5], using a differential forms approach, the constitutive laws that parametrizes all skewonless non-birefringent materials are presented. However, some classes of dispersive NBM cannot be directly addressed by this general approach. This is the case of omega and pseudochiral omega media. In [6], it was shown that Euclidean geometric algebra provides an elegant way of looking into anisotropic NBM as a particular case of other general anisotropy cases. Furthermore, bianisotropic media were already addressed using the same framework [7]. With this formulation a new geometric picture of the interrelation between electric and magnetic anisotropies with the electromagnetic coupling was obtained. In this paper, non-birefringent omega-like media are analyzed using this geometric approach. We derived the necessary conditions to have omega NBM, which are in fact similar to the ones of the anisotropic NBM case. Nevertheless, for omega NBM, additional geometrical restrictions between the permittivity and permeability functions and the electromagnetic coupling are required. On the other hand, a different kind of NBM was found for pseudochiral omega media. The non-birefringence is only observed in a plane, hence the designation 2D-NBM. This effect occurs because the isofrequency surfaces of these media intersect in a plane.

ISBN 978-952-67611-2-1

2. Omega-like NBM

Reciprocal bianisotropic media can be characterized by the following constitutive relations

$$\begin{cases} \mathbf{D}c = Y_0\left[\varepsilon(\mathbf{E}) + \bar{\zeta}(\mathbf{B}c)\right] \\ \mathbf{H} = Y_0\left[\zeta(\mathbf{E}) + \mu^{-1}(\mathbf{B}c)\right] \end{cases} \tag{1}$$

where ε and μ are the linear functions that describe the permeability and permittivity of the material, ζ function describes the electromagnetic coupling, $\bar{\zeta}$ is the adjoint function of ζ corresponding to the usual transpose operation of dyadic calculus, and Y_0 is the admittance of free space. Two types of omega-like NBM are addressed. An omega medium can be characterized by [8]

$$\begin{cases} \varepsilon(\mathbf{a}) = \mu_{yy}^{-1}\left(\varepsilon_{xx}\mu_{yy} - \Omega^2\right)\mathbf{a}_x + \mu_{xx}^{-1}\left(\varepsilon_{yy}\mu_{xx} - \Omega^2\right)\mathbf{a}_y + \varepsilon_{zz}\mathbf{a}_z \\ \mu(\mathbf{a}) = \mu_{xx}\mathbf{a}_x + \mu_{yy}\mathbf{a}_y + \mu_{zz}\mathbf{a}_z \\ \zeta(\mathbf{a}) = i\mu_{xx}^{-1}\,\Omega(\mathbf{z}\times\mathbf{a}) \end{cases} \tag{2}$$

If the function $\eta = \varepsilon^{-1}(\mu)$ is uniaxial along $\mathbf{z}$, i.e.

$$\eta(\mathbf{a}) = \varepsilon^{-1}\left[\mu(\mathbf{a})\right] = \alpha\mathbf{a} + \beta(\mathbf{a}\cdot\mathbf{z})\mathbf{z} \tag{3}$$

these media are uniaxial, which corresponds to the restriction $\varepsilon_{xx}/\mu_{xx} = \varepsilon_{yy}/\mu_{yy} = \eta$. Non-birefringence occurs when $\eta = \varepsilon_{zz}/\mu_{zz}$ (Fig. 1)

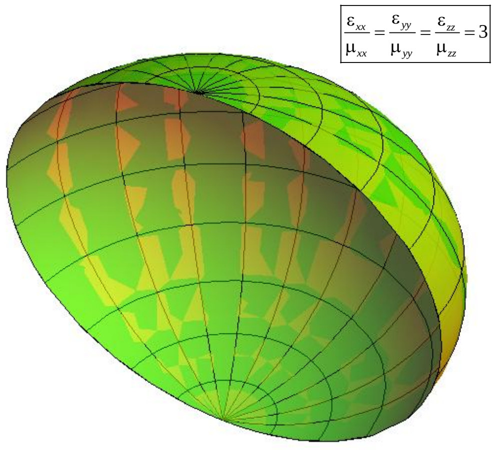

Fig. 1: Isofrequency surfaces of an omega NBM.

On the other hand, pseudochiral omega media are characterized by [8]

$$\begin{cases} \varepsilon(\mathbf{a}) = \varepsilon_{xx}\mathbf{a}_x + \varepsilon_{yy}\mathbf{a}_y + \mu_{xx}^{-1}\left(\mu_{xx}\varepsilon_{zz} - \Omega^2\right)\mathbf{a}_z \\ \mu(\mathbf{a}) = \mu_{xx}\mathbf{a}_x + \mu_{yy}\mathbf{a}_y + \mu_{zz}\mathbf{a}_z \\ \zeta(\mathbf{a}) = -i\mu_{xx}^{-1}\Omega\,(\mathbf{a}\cdot\mathbf{z})\mathbf{x} \end{cases} \tag{4}$$

When $\eta(\mathbf{a}) = \eta\,\mathbf{a}$, we obtain two isofrequency surfaces that intersect along a plane (Fig. 2) where non-birefringent occurs (2D-NBM). This plane corresponds to the kernel of the function ζ. Thus, this medium is non-birefringent when $\zeta(\mathbf{k}) = 0$, where $\mathbf{k}$ is the wave vector direction of the isonormal waves.

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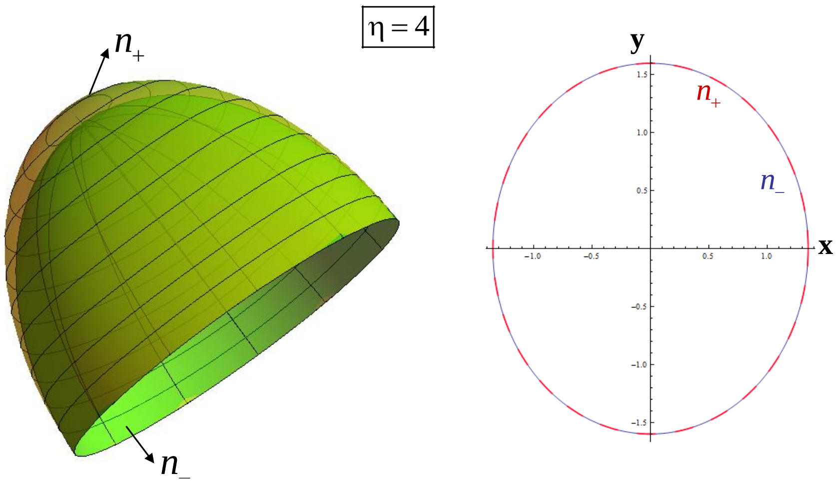

Fig. 1: Isofrequency surfaces of a pseudochiral omega 2D-NBM.

4. Conclusion

In this work we address two types of NBM: i) omega media; ii) pseudochiral omega media. These media are characterized through the impedance like function $\eta = \varepsilon^{-1}(\mu)$ and by the electromagnetic coupling function ζ. We show that for an omega NBM the function η is a uniaxial function along $\mathbf{z}$, where this direction corresponds to the kernel of ζ, i.e. $\zeta(\mathbf{z}) = 0$, and $\eta_1 = \eta_2 = \eta_3 - \Omega^2 / (\varepsilon_{yy}\varepsilon_{xx})$ where η_i are the eigenvalues of the function η. For pseudochiral omega media, we have found a different type of non-birefringence that occurs when the function η is isotropic. When the wave vector belongs to the kernel of ζ, which is a plane, the two isonormal waves have the same phase velocity, i.e. the medium is non-birefringent. However, for other directions of the wave vector, two different refractive indices are obtained. We have dubbed these media as 2D-NBM.

Acknowledgment

This work was partially funded by FCT (Fundação para a Ciência e a Tecnologia), Portugal, under the project PEst-OE/EEI/LA0008/2011.

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