Dirac-cone physics and zero-refractive index medium

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Abstract

Two dimensional (2D) photonic and acoustic/elastic crystals can be designed to exhibit Dirac cone dispersions at the centre of the Brillouin zone at a finite frequency by accidental degeneracy. Using effective medium theories, we can map a subset of these structures to a material with effective permittivity and permeability simultaneously equal to zero in photonic crystals, and with effective mass density and reciprocal of bulk modulus ($1/ C_{44}$) simultaneously equal to zero in acoustic(elastic) crystals. The concept of 2D Dirac cone can be extended to three dimensions.

1. Introduction

The Dirac cones in graphene [1] or photonic/phononic crystals [2] with a honeycomb or triangular lattice have stimulated a lot of interest in simulating relativistic particle behaviours using condensed-matter systems, such as the Klein paradox and Zitterbewegung effect [1-2]. Both the Dirac cones in graphene, photonic crystals and sonic crystals are determined by the intrinsic honeycomb or triangular symmetry of the lattice structure at the Brillouin zone boundary. However, a Dirac cone dispersion at the zone center cannot be obtained by lattice symmetry because time-reversal symmetry dictates the dispersion is generally quadratic at k=0 which excludes the possibility of Dirac cones. On the other hand, the absence of phase variances in the wave transport of the zero-index materials leads to many peculiar wave transport properties, such as the tunnelling electromagnetic waves through subwavelength channels and bends, the tailoring of the radiation phase pattern of arbitrary sources, and cloaking objects inside a channel with specific boundary conditions, and so on [3]. Dirac cone physics and zero-index materials are seeming unrelated but we demonstrate that these two are actually linked together in a subtle way. We realize 2D Dirac cone in photonic [4] and phononic [5-6] systems by accidental degeneracy and then we extend the concept of 2D Dirac cone to 3D case [7]. Using the effective medium [8], we reveal the underlying physics behind the 2D and 3D Dirac cone, and map our structures with homogeneous materials.

2. 2D Dirac cone in photonic and phononic systems

We showed by tuning system parameters, we can realize a triply degenerate state at $k=0$ in the band-structure of a simple 2D photonic crystal consisting of a square lattice of dielectric rods. Two bands have linear dispersions and the bands form Dirac cones that touch at k=0, and an additional flat band will cross the Dirac point frequency. We also obtain the similar band dispersions for EM waves in the triangular lattice. The Dirac cone is induced by accidental degeneracy of monopole and dipole excitations. Using a multiple scattering formalism, we prove the existence of the linear dispersion at $k=0$ and thereby Dirac cones if we have degeneracy of dipole and monopole states. Using standard effective medium theory, we can calculate the effective permittivity and permeability of the photonic crystal, and we find that both of them are simultaneous equal to zero at the Dirac point frequency. This is the case as long as the degeneracy is derived from monopole and dipoles. We demonstrate experimentally

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that the transport properties of the photonic crystal with such Dirac cone do behave as if they were zero-refractive-index materials, such as lensing and cloaking effects in waveguides[4].

For the case of 2D acoustic waves, it can be directly mapped from the 2D photonic crystal system. The effective medium can also be applied to describe this acoustic crystal with both effective mass density and reciprocal of bulk modulus equal to zero [5]. For the case of elastic waves, the physics is different from the EM wave and acoustic wave case. In cubic elastic wave systems, it is well known the dispersions are generally anisotropic, while the dispersions are isotropic in cubic electromagnetic or acoustic systems. In 2D elastic wave systems, there should be two branches of modes, one being quasi-longitudinal and one being quasi-transverse, and the equi-frequency surfaces of these modes are not isotropic, and the system requires 4 parameters (density, C_{11}, C_{12} and C_{44}) to describe the dispersion. We show that in an elastic wave crystal with Dirac cone at $k=0$, the dispersion is "apparently isotropic", meaning that the equi-frequency surfaces are circular, but the eigenmodes are "super-anisotropic" [9] so that the modes are purely longitudinal along some direction and purely transverse along other directions. This type of peculiar behavior, in which the polarization of the elastic-wave is "tied" to the *k*-vector and rotates as *k*-vector sweep around the Brillouin zone is very special. In addition, we showed that the elastic wave crystal have elastic wave effective constants C_{11} and C_{12} that are nearly equal and opposite near the Dirac cone frequency. This peculiar property implies that the elastic wave properties in the Dirac cone is determined only by two parameters (density and C_{44}), instead of four constants (density, C_{11}, C_{12} and C_{44}). Finally, we show that such material can be mapped to an elastic wave material with effectively zero refractive index but a finite group velocity [6].

3. 3D Dirac cone in photonic and phononic systems

We should note here that the notion of Dirac "cone" dispersion is a 2D concept. The equi-frequency surfaces corresponding to 2D Dirac cone dispersion is a set of circles whose radii decrease linearly and approach zero both from above and below the Dirac point. The Dirac point is the frequency at which the equi-frequency circle has zero radius. To obtain a Dirac cone, the dispersion must be linear near the zone center which requires accidental degeneracy. When concept can be generalized from 2D to 3D, the equi-frequency trajectories change from circles in 2D to spheres in 3D. Near the "Dirac point" in 3D systems, the radius of equi-frequency spheres is linearly proportional to $(\omega-\omega_d)$ where ω_d is the Dirac point frequency, and the Dirac point is the frequency at which the equi-frequency sphere becomes a point. The necessary condition is again linear dispersions near the zone center which requires accidental degeneracy. We show that a simple cubic photonic crystal composing of core-shell spheres exhibits a 3D "Dirac point" at the center of Brillouin zone at a finite frequency. The Dirac point is six-fold degenerate and is formed by the accidental degeneracy of electric dipole and magnetic dipole excitations, each with three degrees of freedom. Using effective medium theory, we can map our structure to an isotropic zero-refractive-index material in which all components of the effective permittivity and permeability tensors are simultaneously zero at the Dirac point frequency. Extending the idea from photonic system to acoustic system, we found that 3D Dirac points at $k=0$ can also be found in simple cubic acoustic wave crystals, but different from the case in the photonic system, the 3D Dirac point in acoustic wave system is four-fold degenerate formed by the accidental degeneracy of dipole and monopole excitations. Using the tight-binding method introduced by Sakoda, we can prove the existence of these linear dispersions both in the photonic and acoustic wave systems [10]. Then, using effective medium theory, we can also describe this acoustic wave system as a material which has both effective mass density and reciprocal of bulk modulus equal to zero at the Dirac point frequency [7].

4. Conclusion

We realize 2D Dirac cones at the centre of the Brillouin zone at a finite frequency in photonic and phononic systems by accidental degeneracy. The Dirac cone physics in photonic (acoustic wave) and elastic wave systems are different from each other. We then extend the notion of 2D Dirac point to 3D. Using the effective medium, we can map some of these 2D and 3D systems to a material which has effectively zero permittivity and permeability in photonic system, and has both effective mass density and reciprocal of bulk modulus ($1/C_{44}$) equal to zero in acoustic (elastic) systems at the Dirac point frequency. These "double zero" materials have many interesting wave manipulation properties, such as lensing and cloaking effects.

References

[1] A.H.C. Neto, F. Guinea, N.M.R. Peres, K.S. Novoselov, and A.K. Geim, The electronic properties of grapheme, *Rev. Mod. Phys.*, vol. 81, pp. 109-162, 2009.

[2] M. Plihal, A.A. Maradudin, Photonic band structure of a two-dimensional system: The triangular lattice, *Phys. Rev. B*, vol. 44, p. 8565, 1991; R.A. Sepkhanov, Y.B. Bazaliy, C.W.J. Beenakker, Extremal transmission at the Dirac point of a photonic band structure, *Phys. Rev. A*, vol. 75, p. 063813, 2007; F.D.M. Haldane, S. Raghu, Possible realization of directional optical waveguides in photonic crystals with broken time-reversal symmetry, *Phys. Rev. Lett.*, vol, 100, p. 013904, 2008; X. Zhang, Observing zitterbewegung for photons near the Dirac point of a two dimensional photonic crystal, *Phys. Rev. Lett.*, vol. 100, p. 113903, 2008; X. Zhang, Z. Liu, Extremal transmission and beating effect of acoustic waves in two-dimensional sonic crystals, *Phys. Rev. Lett.*, vol. 101, p. 264303, 2008.

[3] M. Silveirinha, N. Engheta, Tunneling of electromagnetic energy through subwavelength channels and bends using ε -near-zero materials, *Phys. Rev. Lett.*, vol. 97, p. 157403, 2006; A. Alu, M.G. Silveirinha, A. Salandrino, N. Engheta, Epsilon-near-zero metamaterials and electromagnetic sources: tailoring the radiation phase pattern, *Phys. Rev. B*, vol. 75, p. 155410, 2007; J. Hao, W. Yan, M. Qiu, Super-reflection and cloaking based on zero index metamaterial, *Appl. Phys. Lett.*, vol. 96, p. 101109, 2010; R.W. Ziolkowski, Propagation in and scattering from a matched metamaterial having a zero index of refraction, *Phys. Rev. E*, vol. 70, p. 046608, 2004.

[4] X. Huang, Y. Lai, Z.H. Hang, H. Zheng, C.T. Chan, Dirac cones induced by accidental degeneracy in photonic crystals and zero-refractive-index materials, *Nature Materials*, vol. 10, p. 582, 2011.

[5] F. Liu, X. Huang, C.T. Chan, Dirac cones at $k=0$ in acoustic crystals and zero refractive index acoustic materials, *Appl. Phys. Lett.*, vol. 100, p. 071911, 2012.

[6] F. Liu, Y. Lai, X. Huang, C.T. Chan, Dirac cones at $k = 0$ in phononic crystals, *Phys. Rev. B*, vol. 84, 224113, 2011.

[7] X. Huang, F. Liu, C.T. Chan, *to be submitted.*

[8] Y. Wu, J. Li, Z.Q. Zhang, C.T. Chan, Effective medium theory for magnetodielectric composites: beyond the long-wavelength limit, *Phys. Rev. B*, vol. 74, 085111, 2006; Y. Wu, Z.Q. Zhang, Dispersion relations and their symmetry properties of electromagnetic and elastic metamaterials in two dimensions, *Phys. Rev. B*, vol. 79, p. 195111, 2009.

[9] Y. Lai, Y. Wu, P. Sheng, Z. Q. Zhang, Hybrid elastic solids, *Nature Materials*, vol. 10, p. 620, 2011.

[10] K. Sakoda, Dirac cone in two- and three-dimensional metamaterials, *Opt. Express*, vol. 20, pp. 3898–3917, 2012.

