Small metal particles as resonators for microwave, terahertz and optical frequencies

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Abstract

In this paper we analyze resonance response of single split-ring and core-shell resonators whose sizes vary from few millimeters to tens of nanometers to external electromagnetic field in wide spectral range spanning from microwave to optics. Polarizability spectra, quality factors and resonance frequencies were obtained using numerical simulation. The limits of scaling were determined for parameters of dipole polarizability.

1. Introduction

Small metal particles are widely and successfully used as resonant elements for optical antennas [1], nanolasers [2] and as basic building blocks in the design of metamaterials [3, 4]. For the recent decade operating frequencies of metamaterials have increased from microwave to visible range, consequently dimensions of resonant particles have decreased from few centimeters to tens of nanometers. So it is of great interest to know how do polarizability parameters of a metal resonator depend on it's dimensions, with account of the fact that complex permittivity of a metal in such wide spectral range is experiencing strong dispersion. Some studies were made in this direction [5, 6], but they were limited to finding patterns of behavior of the resonance frequencies and did not include the study of quality factors and amplitudes of resonances of polarizabilities when particles were resized. We conducted more detailed research of dimension dependencies using numerical simulation. Two types of objects were considered: gold split-ring resonators (SRR) and gold spherical core-shell resonators (CSR) ($n_{shell} = 1.5$). These particles are, on the one hand, widely used in the field of metamaterial technology, and, on the other hand, significantly differ from each other in their symmetry and geometry. Complex permittivity of gold was taken from [7]. Time dependence $\exp(-i\omega t)$ was assumed in calculations.

2. Polarizability spectra, resonance frequencies and quality factors

The polarizability of an arbitrary particle is described in general case by four second-order tensors: $\hat{\alpha}^e$, $\hat{\alpha}^m$, $\hat{\alpha}^{em}$ and $\hat{\alpha}^{me}$ being electric, magnetic and pair of magnetoelectric polarizabilities correspondingly. The contribution of each polarizability in general electromagnetic response of the particle depends on its shape, dimensions, material and the external wavelength. These tensors are included in linear relations between complex amplitudes of electric $\vec{p}$ and magnetic $\vec{m}$ dipole moments of the particle and the external fields $\vec{E}$ and $\vec{H}$ which induced these moments:

$$\begin{aligned} \vec{p} &= \hat{\alpha}^e \varepsilon_0 \vec{E} + \hat{\alpha}^{em} \varepsilon_0 Z \vec{H}, \\ \vec{m} &= \hat{\alpha}^{me} Z^{-1} \vec{E} + \hat{\alpha}^m \vec{H}, \end{aligned} \tag{1}$$

where $Z = \sqrt{\mu_0/\varepsilon_0}$ – the impedance of free space. We used the polarizability matrix calculation method described in [8].

For the sake of simplicity and comparison the values of polarizabilities were normalized to the particles' volumes V (l^2h for SRR and $\pi D^3/6$ for CSR). Results of calculation of electric polarizability spectra are shown on Fig. 1. Similar frequency dependencies were also obtained for magnetic and magnetoelectric polarizabilties. Main LC-resonance of SRR (Fig. 1b) corresponds to mode with circular motion of conductivity current in the metal and displacement current in the capacitive gap. Another higher-frequency plasmon resonance corresponds to current oscillation along the base of SRR. The contribution of other modes in polarizability of SRR is negligibly small. In contrast, CSR (Fig. 1c) have only modes with separate electric or magnetic response (Mie resonances).

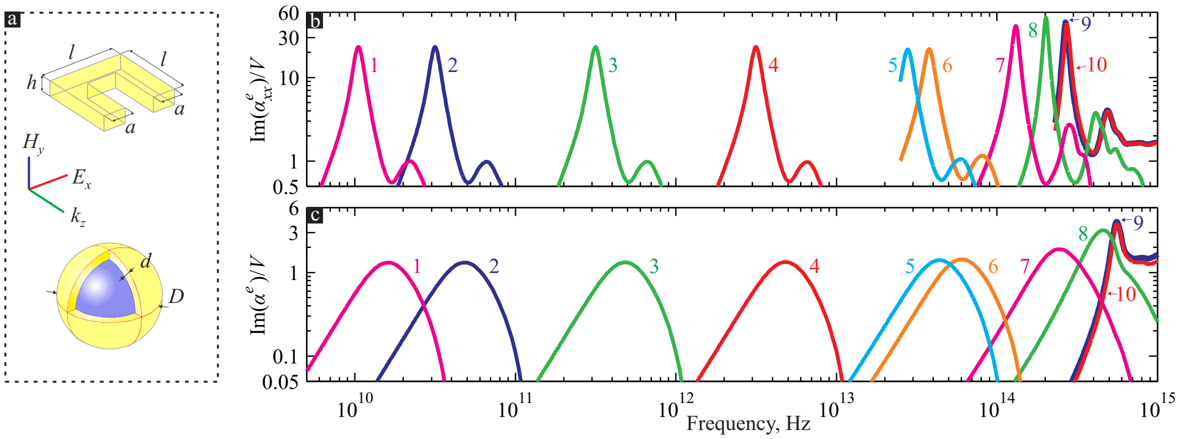

Fig. 1: Test particles, its dimensions and external field polarization (a). Imaginary part of normalized electric polarizability spectra for SRR (b) and CSR (c). The numbers corresponds to particles sizes $l\times h$ and $D\times d$: 1 – 6×1.2 mm, 2 – 2×0.4 mm, 3 – 200×40 μm, 4 – 20×4 μm, 5 – 2.2×0.44 μm, 6 – 1.6×0.32 μm, 7 – 400×80 nm, 8 – 200×40 nm, 9 – 50×10 nm, 10 – 25×5 nm.

The regularities of spectral and dimensional behavior demonstrate agreement with the concept of localized plasmon-polariton nature of the oscillations of small metal particles. For microwave to mid-IR frequencies the main part of coupled oscillations energy is concentrated in the electromagnetic field around the particle, whereas the energy stored at skin depth of metal is just small portion of its total amount. Hence the main losses are due to evanescent waves. Since the linear size of the resonators has the same order as the wavelength, the radiation intensity is sufficiently high, respectively quality factor is low. As long as surrounding space of the particle has no dispersion, simultaneous change of the wavelength and dimensions of the resonator leads to proportional alteration in spatial distribution of the electromagnetic field. Thus, an electrodynamic scaling takes place, which means that the resonance frequency is proportional to inverted linear size of the particle, i.e. $\nu \sim s^{-1}$, where $s = \{l, D\}$ (Fig. 2a), the quality factor is independent on the frequency (Fig. 2b), and the normalized polarizability is constant from microwave to mid-IR range (Fig. 1, curves 1 – 6). As a consequence, if all dimensions of a metamaterial and a wavelength are equally reduced, then in the limits of scaling law, its transmission and reflection coefficients will remain unchanged.

When approaching optical region, drastic redistribution of energy occurs between the evanescent waves in the vicinity of the resonator and the plasma oscillations in the metal. This redistribution is due to growth of the skin depth, which is accompanied by a drop of energy that radiates into outer space. Consequently, higher frequncies means bigger portion of the energy localized in the metal in the form of kinetic energy of the oscillations of the electron subsystem, additionally imaginary part of the permittivity quickly decreases, hence lower losses and higher quality factor (Fig. 2b). But close proximity of the Langmuir plasma frequency leads to saturation of resonance frequency (Fig. 2a). Dependence of resonance frequency ν_{res} from size of a resonator s can be described by following scaling law [5]:

$$\nu_{res} = 1/\sqrt{\text{const} \cdot s^2 + \nu_{max}^2} \tag{2}$$

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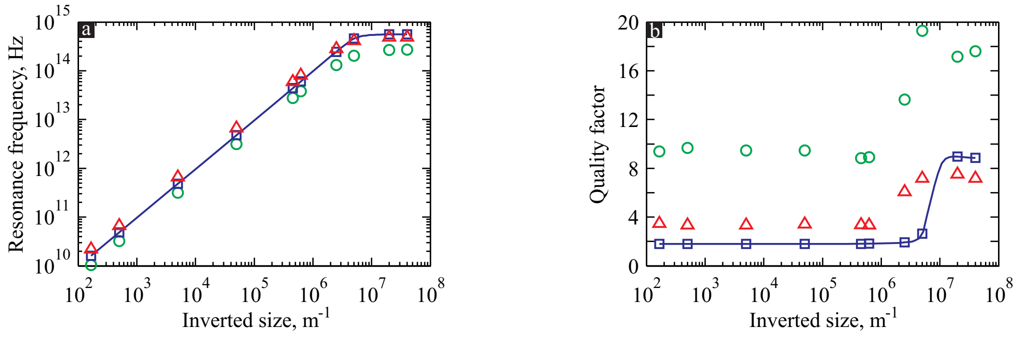

Fig. 2: Resonance frequency (a) and quality factor (b) versus inverted size s^{-1}. Circles – LC-resonances, triangles – plasmon resonances of SRR, solid line with squares – Mie resonances of CSR

3. Conclusion

The presented results demonstrate that small metal particles of different shape obey similar dimentional behavior of resonance frequencies corresponding to the law (2), which is also valid for different oscillation modes. Though quality factors and polarizabilities highlighted here were established for the two types of particles by numerical simulation these regularities seem to have more general nature.

Electrodynamic scaling law also takes place for quality factors of particles with resonance frequencies from microwave to mid-IR range. This allows, in particular, to facilitate the design of metamaterials and devices with metal particles for terahertz and far-IR ranges, using not only the results of electrodynamic calculations, but also full-scale experiments at microwaves. Thus the number of experiments carried out in the "inconvenient" frequency ranges may be significantly reduced.

Larger quality factors can be achieved with resonators in optical spectrum. Addition opportunities may bring use of silver instead gold and operation at low temperatures. It should be noted that the observation of resonant response of metal nanoparticles could allow measurements of complex permittivity of a metal with higher accuracy than the customary method of measurement based on light reflection from a metal film.

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